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Two-Photon Entanglement in a Two-Mode Supersymmetric Model

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We will study entangled two-photon states generated from a two-mode supersymmetric model and quantify degree of entanglement in terms of the entropy of entanglement. The influences of the nonlinearity on the degree of entanglement is also examined, and it is shown that amount of entanglement increase with increasing the nonlinear coupling constant.

KEY WORDS: entanglement; supersymmetry; Higgs algebra.

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1. INTRODUCTION

Perhaps, quantum entanglement is the most nonclassical features of quantum mechanics which has recently attracted much attention although it was discovered many decades ago by Einstein, Podolsky, and Rosen (1935) and SchrÖdinger (1935). It plays a central role in quantum information theory and provides potential resources for communication and information processing (Bennett and Wiesner, 1992; Bennett *et al.*, 1993; Bennett *et al.*, 1996). By definition, a pure quantum state of two or more subsystems is said to be entangled if it is not a product of states of each components. A lot of works have been devoted to the preparation and measurement of entangled states. Moreover the possibility for generation of the entangled states with a fixed photon number has been theoretically studied (Duan *et al.*, 2000a,b; Cochrane *et al.*, 2000; Liu *et al.*, 2004). Duan *et al.*, described an entanglement purification protocol which generates maximally entangled states with fixed photon number from squeezed vacuum states or from mixed Gaussian continuous states by the quantum nondemolition measurement (Duan *et al.*, 2000a,b). Quantum teleportation using an entangled source of fixed photon number has also been theoretically investigated in (Cochrane, 2000). Liu *et al.*, are used a system of two coupled microcrytallites as a source with fixed exciton number and

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quantified entanglement of the excitonic states (Liu *et al.*, 2004). Therefore, the generation of a new entangled source with fixed photon number is an interesting task both from experimental and theoretical viewpoints.

In this contribution, it is shown that a two-mode field with a two-photon interaction can be used as a good source for generation of entangled states with fixed photon number. We will study entangled states generated from two degenerate bosonic systems with fixed photon number, and we concern on quadratic nonlinearity between modes to use Higgs algebra as the spectrum generating algebra of the corresponding Hamiltonian (Debergh, 1998; Beckers, 1999). We also restrict ourselves to the case that total number of photons is odd. For this case, Debergh in (Debergh, 1998) have shown that the corresponding Hamiltonian is supersymmetric (Witten, 1981).

A number of entanglement measures have been discussed in the literature, such as the von Neumann reduced entropy, the relative entropy of entanglement (Plenio and Vdral, 1998) and the so called entanglement of formation (Bennett *et al.*, 1996). In order to discus entanglement of the states, we use von Neumann reduced entropy which has widely been accepted as an entanglement measure for pure bipartite states.

The organization of the paper is as follows. In Section 2 we introduce a quantum optics model for two bosonic system with supersymmetric feature. An analytial solution of the Hamiltonian is also given by following the method of (Debergh, 1998). In Section 3, the analytical results of Section 2 are employed to generate entangled two-photon states with fixed photon number. Some examples are also considered in Section 3. The paper is concluded in Section 4 with a brief conclusion.

2. THE TWO-MODE SUPERSYMMETRIC HAMILTONIAN

In this section, we shall introduce and analyse a model for nonlinear interaction between two-mode field. Our method is based on the analysis given by (Debergh, 1998). Let us consider the following family of Karrassiov–Klimov Hamiltonian (Karassiov and Klimov, 1994) which describes multi-photon process of scattering, i.e.,

$$H = \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2 + g(a_1^\dagger)^s a_2^r + g^* a_1^s (a_2^\dagger)^r \tag{1}$$

where $0 \leq r \leq s$, g is coupling constant and ω_i $(i = 1, 2)$ refer to angular frequencies of two-mode field characterized by annihilation and creation operators $a_i, a_i^\dagger$ respectively, satisfying $[a_i, a_j^\dagger] = \delta_{ij}$. Hamiltonian (1) can be rewritten as

$$H = (\omega_1 + \omega_2)R_0 + (s\omega_1 - r\omega_2)J_0 + gJ_+ + g^* J_-, \tag{2}$$

where (Beckers *et al.*, 1999)

$$R_0 \equiv \frac{1}{r+s}(r a_1^\dagger a_1 + s a_2^\dagger a_2), \tag{3}$$

and

$$J_0 \equiv \frac{1}{r+s}(a_1^\dagger a_1 - a_2^\dagger a_2), \qquad J_+ \equiv (a_1^\dagger)^s a_2^r, \qquad J_- \equiv a_1^s (a_2^\dagger)^r. \tag{4}$$

It can be easily shown that

$$[R_0, J_0] = [R_0, J_\pm] = 0, \tag{5}$$

and

$$[J_0, J_\pm] = \pm J_\pm, \tag{6}$$

for arbitrary values of r and s.

It is obvious that R_0 is a constant of motion, and the total photon number of the two-mode system is conserved. Moreover, the infinite dimensional vectors $\{|n_1, n_2\rangle = \frac{(a_1^\dagger)^{n_1}(a_2^\dagger)^{n_2}}{\sqrt{n_1!n_2!}}|0,0\rangle, n_1, n_2 = 0, 1, 2, \cdots\}$ are eigenvectors of R_0 with corresponding eigenvalues $j = \frac{rn_1+sn_2}{r+s}$. Debergh (1998) has shown that in order to have Higgs algebra as the spectrum generating algebra of the Hamiltonian (2), we have to add to (6) the following requirement

$$[J_+, J_-] = 2J_0 + 8\beta J_0^3, \tag{7}$$

and have shown that (Debergh, 1998; Beckers, 1999) this is possible only for $r = s = 2$, with parameter β given by

$$\beta = -\frac{4}{4j^2 + 4j - 2}, \qquad j = 0, \frac{1}{2}, 1, \ldots. \tag{8}$$

These values of β lead to the relations (Debergh, 1997)

$$J_3|j,m\rangle = \frac{m}{2}|j,m\rangle, \tag{9}$$

$$J_\pm|j,m\rangle = \sqrt{(j \mp m)(j \pm m + 1)(j \mp m - 1)(j \pm m + 2)}|j, m \pm 2\rangle, \tag{10}$$

for $m = -j, -j+1 \cdots, +j$. For a fixed total photon number N, the vectors $|j,m\rangle$ are related to the two-mode Fock states by

$$|j,m\rangle = |m_1\rangle_A |m_2\rangle_B = \frac{(a_1^\dagger)^{j+m}(a_2^\dagger)^{j-m}}{\sqrt{(j+m)!(j-m)!}}|0\rangle_A|0\rangle_B, \tag{11}$$

where $|m_1\rangle_A \otimes |m_2\rangle_B$ represent Fock state with $m_1 = j + m$ photons in mode A and $m_2 = j - m$ photons in mode B.

In order to have more symmetry in Hamiltonian (2), let us suppose that $\omega_1 = \omega_2 = \omega$, and concern on the case that g is real. In this case, Hamiltonian (1) reduce to

$$\begin{aligned} H &= \omega\big(a_1^\dagger a_1 + a_2^\dagger a_2\big) + g\big(\big(a_1^\dagger\big)^2 a_2^2 + a_1^2\big(a_2^\dagger\big)^2\big) \\ &= 2\omega R_0 + g\,(J_+ + J_-)\,. \end{aligned} \tag{12}$$

Now, by expanding eigenvectors of (12) as $|\psi_k\rangle = \sum_{m=-j}^{m=j} C_m^{(k)}|j, m\rangle$ and using eigenvalue equation $H|\psi_k\rangle = E_k|\psi_k\rangle$, we get

$$\begin{aligned} E_k C_m^{(k)} &= 2j\omega C_m^{(k)} + gC_{m-2}^{(k)}\sqrt{(j+m)(j+m-1)(j-m+1)(j-m+2)} \\ &\quad + gC_{m+2}^{(k)}\sqrt{(j-m)(j-m-1)(j+m+1)(j+m+2)}. \end{aligned} \tag{13}$$

Moreover in order to have supersymmetric Hamiltonian, Debergh concerned on the case that j is a half-integer, which leads to two fold degeneracy of all eigenenergies as

$$E_k = 2\omega j + g\lambda_k, \qquad k = 1, 2, \cdots, j + \frac{1}{2}, \tag{14}$$

where λ_k is anyone of the $j + \frac{1}{2}$ different solutions of (Debergh, 1998)

$$\begin{aligned} [F(A_k, j, \lambda)]^2 \equiv \Bigg[& \lambda^{j+\frac{1}{2}} - \sum_{k=1}^{j-\frac{1}{2}} A_k^2 \lambda^{j-\frac{3}{2}} + \left(\sum_{k<l,\ |k-l|\neq 2}^{j-\frac{1}{2}} A_k^2 A_l^2 - A_{j-\frac{3}{2}}^2 A_{j-\frac{1}{2}}^2 \right) \lambda^{j-\frac{7}{2}} \\ & - \left(\sum_{k<l<p,\ |k-l|\neq 2,\ |k-p|\neq 2,\ |l-p|\neq 2}^{j-\frac{1}{2}} A_k^2 A_l^2 A_p^2 - \sum_{k=1}^{j-\frac{9}{2}} A_k^2 A_{j-\frac{3}{2}}^2 A_{j-\frac{1}{2}}^2 \right) \lambda^{j-\frac{11}{2}} \cdots \Bigg]^2, \end{aligned} \tag{15}$$

where A_k are defined by

$$A_k = (k(k+1)(2j-k)(2j-k+1))^{\frac{1}{2}}, \qquad k = 1, 2, \cdots, j - \frac{1}{2}. \tag{16}$$

Let us denote two eigenvectors of H corresponding to twofold degenerate eigenvalue E_k with $|\psi_k^{(1)}\rangle$ and $|\psi_k^{(2)}\rangle$. Now, since (13) relates coefficient $C_m^{(k)}$ to $C_{m+2}^{(k)}$ and $C_{m-2}^{(k)}$, we can, without los of generality, write these two orthonormal eigenvectors belonging to eigensubspace ε_k as

$$\begin{aligned} \big|\psi_k^{(1)}\big\rangle &= \sum_{n=0}^{j-\frac{1}{2}} C_{j-2n}^{(k)}|j, j-2n\rangle, \quad C_{j-2n}^{(k)} = \frac{b_{j-2n}^{(k)}}{\sqrt{\sum_{n=0}^{j-\frac{1}{2}}\left(b_{j-2n}^{(k)}\right)^2}}, \\ \big|\psi_k^{(2)}\big\rangle &= \sum_{n=0}^{j-\frac{1}{2}} C_{j-2n-1}^{(k)}|j, j-2n-1\rangle, \quad C_{j-2n-1}^{(k)} = \frac{b_{j-2n-1}^{(k)}}{\sqrt{\sum_{n=0}^{j-\frac{1}{2}}\left(b_{j-2n-1}^{(k)}\right)^2}}, \end{aligned} \tag{17}$$

where

$$b_j^{(k)} = 1, \quad b_{j-2n}^{(k)} = \frac{F(A_{2p-1}, n - \frac{1}{2}, \lambda_k)}{A_1 A_3 \cdots A_{2n-1}}, \quad n = 1, \ldots, j - \frac{1}{2},$$
$$b_{j-1}^{(k)} = 1, \quad b_{j-2n-1}^{(k)} = \frac{F(A_{2p}, n - \frac{1}{2}, \lambda_k)}{A_2 A_4 \cdots A_{2n}}, \quad n = 1, \cdots, j - \frac{1}{2}, \tag{18}$$

where function F has been defined in (15). In two-mode Fock space representation, (17) can be written as

$$\left|\psi_k^{(1)}\right\rangle = \sum_{n=0}^{\frac{2j-1}{2}} C_{j-2n}^{(k)} |2j - 2n\rangle_A |2n\rangle_B,$$
$$\left|\psi_k^{(2)}\right\rangle = \sum_{n=0}^{\frac{2j-1}{2}} C_{j-2n}^{(k)} |2j - 2n - 1\rangle_A |2n + 1\rangle_B. \tag{19}$$

Finally, evolution operator $U(t)$ takes the following form

$$U(t) = \sum_{k=1}^{j+\frac{1}{2}} e^{-iE_k t} \left(\left|\psi_k^{(1)}\right\rangle\!\left\langle\psi_k^{(1)}\right| + \left|\psi_k^{(2)}\right\rangle\!\left\langle\psi_k^{(2)}\right| \right). \tag{20}$$

3. TWO-PHOTON ENTANGLEMENT

In this section, we will study entangled states generated by Hamiltonian (12). The entanglement measure that we are going to use is, the so called von Neumann entropy of reduced density matrix, which has most widely been accepted as an entanglement measure of pure state of a bipartite system. Let $|\psi\rangle$ be a pure state of a bipartite system with state space $H_A \otimes H_B$. Entanglement of $|\psi\rangle$ is defined by

$$E(|\psi\rangle) = -\mathrm{Tr}(\rho_A \ln \rho_A) = -\mathrm{Tr}(\rho_B \ln \rho_B) = \sum_n \lambda_n^2 \ln \lambda_n^2, \tag{21}$$

where ρ_A is reduced density matrix of subsystem A which is obtained by tracing out subsystem B, i.e., $\rho_A = \mathrm{Tr}_B(|\psi\rangle\langle\psi|)$, ρ_B is defined similarly, and λ_n are square root of nonzero eigenvalues of ρ_A and ρ_B. They are also Schmidt number of state $|\psi\rangle$, i.e.,

$$|\psi\rangle = \sum_n \lambda_n |u_n\rangle_A |v_n\rangle_B, \tag{22}$$

where $\{|u_n\rangle\}$ and $\{|v_n\rangle\}$ are orthonormal states of two subsystems A and B, respectively. The definition is based on the fact that although entropy of a pure state is zero, but von Neumann entropy of each subsystem is zero only when the state $|\psi\rangle$ is a product state.

In this paper, we shall consider the case that the total number of photons in the whole system is fixed by the initial condition $N = 2j$, and system is initially in product state

$$|\psi(0)\rangle = |N - L\rangle_A |L\rangle_B, \tag{23}$$

which represents initially $N - L$ photons in mode A and L photons in mode B. By taking account of (19), (20), (23), we obtain, up to an overall phase factor $e^{iN\omega}$, the final state of the system by

$$|\psi^{(N-L,L)}(t)\rangle = \sum_{n=0}^{\frac{N-1}{2}} a_n |N - 2n - \Delta_L\rangle_A |2n + \Delta_L\rangle_B, \tag{24}$$

where coefficients a_n are defined by

$$a_n = \sum_{k=1}^{\frac{N+1}{2}} e^{-i\lambda_k t}\ C^{(k)}_{\frac{N}{2}-L} C^{(k)}_{\frac{N}{2}-2n-\Delta_L}, \tag{25}$$

and Δ_L is difined such that it is zero (one) when L is an even (odd) integer. Obviously, (24) represents final state of the system in Schmidt form and, accordingly, the von Neumann entropy of the reduced density matrix can be obtained easily by

$$E^{(N-L,L)}(t) = -\sum_{n=0}^{j-\frac{1}{2}} |a_n|^2 \ln |a_n|^2. \tag{26}$$

Finally, it should be stressed that according to (26) maximal entangled state of a system with the total photon number N is

$$\left|\psi^{(N)}_{\text{MAX}}\right\rangle = \frac{1}{\sqrt{N+1}} \sum_{n=0}^{N} |N - n\rangle_A |n\rangle_B, \tag{27}$$

where in this case entropy of entanglement is equal to $E^{(N)}_{\text{MAX}} = \ln(N+1)$. On the other hand, for state given by (24), maximum entropy of entanglement is obtained when $a_n = \sqrt{\frac{2}{N+1}}$, i.e.,

$$\left|\psi^{(N-L,L)}_{\text{MAX}}\right\rangle = \sqrt{\frac{2}{N+1}} \sum_{n=0}^{\frac{N-1}{2}} |N - 2n - \Delta_L\rangle_A |2n + \Delta_L\rangle_B, \tag{28}$$

where we find $E^{(N-L,L)}_{\text{MAX}} = \ln(\frac{N+1}{2})$. This means that for a system with fixed photon number N, maximum entanglement that can be achieved from Hamiltonian (12) is less than maximum entanglement that can be obtained from a system that linear interaction between modes is also considered. The difference between these two maximum is, of course, constant and equal to ln2.

In the rest of this section, we will consider some examples in $N = 1, 3, 5, 9$ and discuss results.

1. $\mathbf{j} = \frac{1}{2}$. In this case, the total number of photons of system is 1, and because of the two fold degeneracy of eigenvalues, the whole state space of system coincide with eigensubspace of the only eigenvalue. Accordingly the final state $|\psi(t)\rangle$ differs with initial product state only in a total phase factor, therefore, we cannot have entanglement.
2. $\mathbf{j} = \frac{3}{2}$. In this case, the state space of system decomposes into two eigensubspces, with eigenvalues
$$E_1 = 3\omega + \sqrt{12}g \qquad E_2 = 3\omega - \sqrt{12}g. \tag{29}$$
By starting with two initial states $|\psi(0)\rangle = |3\rangle_A|0\rangle_B$ and $|\psi(0)\rangle = |2\rangle_A|1\rangle_B$ we obtain, respectively
$$|\psi^{(3,0)}(t)\rangle = \cos(\sqrt{12}gt)|3\rangle_A|0\rangle_B - i\sin(\sqrt{12}gt)|1\rangle_A|2\rangle_B, \tag{30}$$
$$|\psi^{(2,1)}(t)\rangle = \cos(\sqrt{12}gt)|2\rangle_A|1\rangle_B - i\ \sin(\sqrt{12}gt)|0\rangle_A|3\rangle_B. \tag{31}$$
By using (26), we obtain same value for entanglement of the above two states as
$$\begin{aligned} E^{(3,0)}(t) = E^{(2,1)}(t) = &- \cos^2(\sqrt{12}gt)\ \ln(\cos^2(\sqrt{12}gt)) \\ &- \sin^2(\sqrt{12}gt)\ \ln(\sin^2(\sqrt{12}gt)). \end{aligned} \tag{32}$$
Equation (32) shows that entanglement has zero value when $t = \frac{k\pi}{2g}$ (for $k = 0, 1, \ldots$) and it takes maximum value ln2 at times $t = \frac{(2k+1)\pi}{4g}$ (for $k = 0, 1, \ldots$). This, obviously, shows that the survival time of maximum entanglement decrease with increasing of the nonlinear coupling constant g.
3. $\mathbf{j} = \frac{5}{2}$. This case corresponds with a system that has five photons and the state space of system decomposes into three eigensubspces, with eigenvalues
$$E_1 = 5\omega \qquad E_2 = 5\omega + 4\sqrt{7}g \qquad E_3 = 5\omega - 4\sqrt{7}g. \tag{33}$$
In this case, by considering the initial state as anyone of $|\psi(0)\rangle = |5\rangle_A|0\rangle_B$, $|\psi(0)\rangle = |4\rangle_A|1\rangle_B$ and $|\psi(0)\rangle = |3\rangle_A|2\rangle_B$, we find, respectively, the final state of the system as
$$\begin{aligned} \left|\psi^{(5,0)}(t)\right\rangle = &\frac{1}{14}\left(9 + 5\cos(4\sqrt{7}gt)\right)|5\rangle_A|0\rangle_B \\ &-i\sqrt{\frac{5}{14}}\sin(4\sqrt{7}gt)|3\rangle_A|2\rangle_B \\ &+\frac{3\sqrt{5}}{14}\left(-1 + \cos(4\sqrt{7}gt)\right)|1\rangle_A|4\rangle_B, \end{aligned} \tag{34}$$

$$|\psi^{(4,1)}(t)\rangle = \frac{1}{14}\left(5+9\cos(4\sqrt{7}gt)\right)|4\rangle_A|1\rangle_B - i\frac{3}{\sqrt{14}}\sin(4\sqrt{7}gt)|2\rangle_A|3\rangle_B + \frac{3\sqrt{5}}{14}\left(-1+\cos(4\sqrt{7}gt)\right)|0\rangle_A|5\rangle_B, \tag{35}$$

$$|\psi^{(3,2)}(t)\rangle = -i\sqrt{\frac{5}{14}}\sin(4\sqrt{7}gt)|5\rangle_A|0\rangle_B + \cos(4\sqrt{7}gt)|3\rangle_A|2\rangle_B - i\frac{3}{\sqrt{14}}\sin(4\sqrt{7}gt)|1\rangle_A|4\rangle_B. \tag{36}$$

Fig. 1 demonstrates the evolution of the entropy of entanglement as a function of gt for three different initial states with different nonlinear coupling constant g. The figure is plotted such that the top horizontal line of each curve corresponds to the maximum entanglement ln3. The maximum entanglement that can be obtained by system is different for different initial state and the system can reach, approximately, to maximum entanglement ln3 only in the case, that the initial state is $|\psi(0)\rangle = |5\rangle_A|0\rangle_B$. The Fig. 1 also shows that the survival time of maximum entanglement decrease when the difference between photon numbers of two modes A and B of the initial state is decreased. As the horizontal axis of the curves is product of coupling constant g and time t, it is obvious that by increasing the nonlinear constant g, survival time of maximum entanglement decreases. Equations (34), (35) and (36) show that if the nonlinear coupling constant g is equal to zero, then $|\psi(t)\rangle = |\psi(0)\rangle$, i.e., we cannot have entangled state.

4. $\mathbf{j} = \frac{9}{2}$. Finally we consider as the last example the system with nine photons and accordingly the state space of system decomposes into five eigensubspces, with eigenvalues

$$E_1 = 9\omega$$
$$E_2 = 9\omega + \sqrt{792+24\sqrt{561}}g \quad E_3 = 9\omega + \sqrt{792-24\sqrt{561}}g \tag{37}$$
$$E_4 = 9\omega - \sqrt{792+24\sqrt{561}}g \quad E_5 = 9\omega - \sqrt{792-24\sqrt{561}}g.$$

The evolution of the entropy of entanglement as a function of gt for four different initial states $|\psi(0)\rangle = |9\rangle_A|0\rangle_B$, $|\psi(0)\rangle = |8\rangle_A|1\rangle_B$, $|\psi(0)\rangle = |6\rangle_A|3\rangle_B$ and $|\psi(0)\rangle = |5\rangle_A|4\rangle_B$ is demonstrated in Fig. 2. The

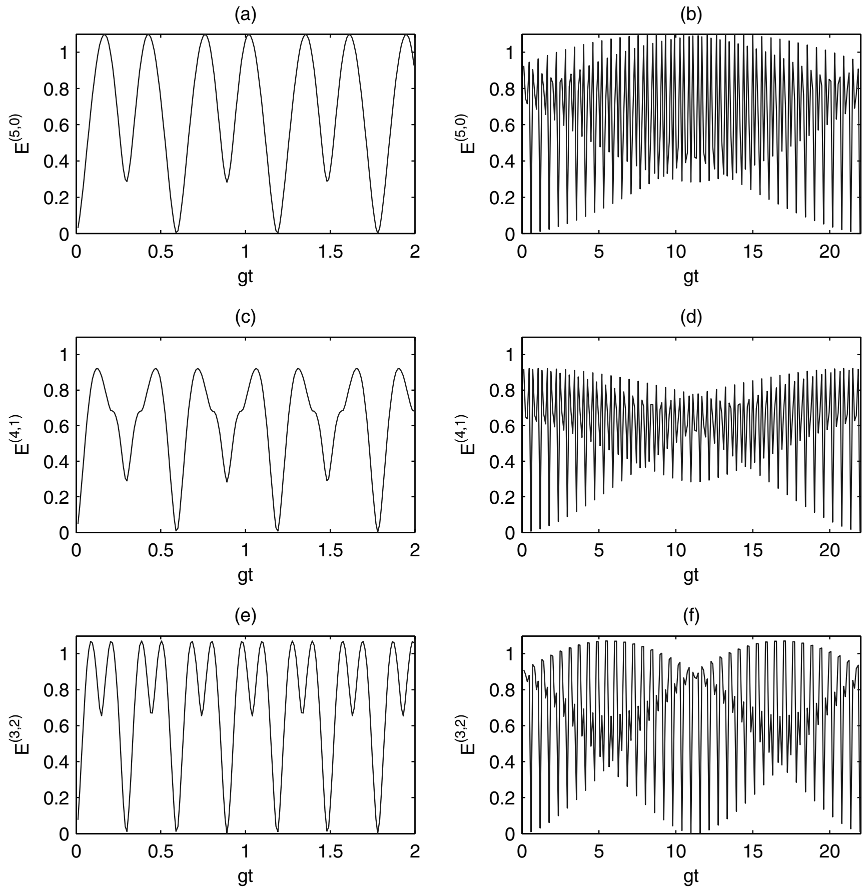

Fig. 1. $E^{(5,0)}$, $E^{(4,1)}$ and $E^{(3,2)}$ are plotted as a function of gt in interval [0, 2] (curves (a), (c) and (e)) and in interval [0, 22.015] (curves (b), (d) and (f)).

maximum entanglement that can be obtained by system is different for different initial states (the top horizontal line of each curve corresponds to the maximum entanglement ln5). We find that the maximum entanglement ln5 is obtained, approximately, only in the case, that there are nine photons initially in one of the modes (for example mode A), i.e., $|\psi(0)\rangle = |9\rangle_A|0\rangle_B$. The survival time of the maximum entanglement decrease by increasing the nonlinear constant g and it is also decrease by decreasing the difference between photon number of two modes A and B of the initial state.

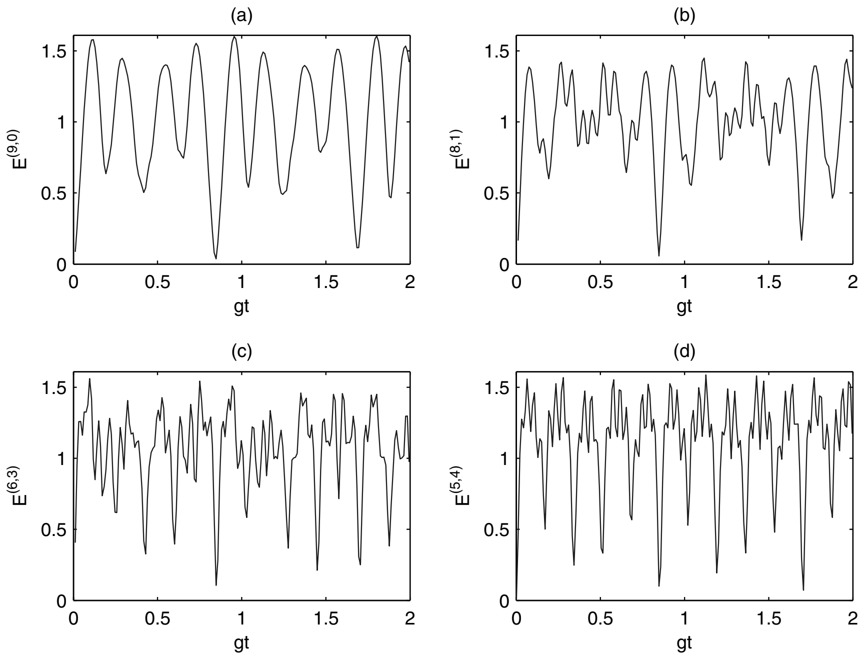

Fig. 2. $E^{(9,0)}$, $E^{(8,1)}$, $E^{(6,3)}$ and $E^{(5,4)}$ are plotted as a function of gt (curves (a), (b), (c) and (d)).

4. CONCLUSION

We studied entangled states generated from two-mode supersymmetric model with fixed photon number. We found that only in the case, that system has $N = 3$ photons, the maximum entanglement can be obtained exactly. For other systems with total photon number greater than three, we found that the maximum entanglement is obtained, approximately, only in the case, that all photons are initially in one of the modes, i.e., $|\psi(0)\rangle = |N\rangle_A|0\rangle_B$ or $|\psi(0)\rangle = |0\rangle_A|N\rangle_B$. The influences of the nonlinearity on the degree of entanglement is also examined, and is shown that survival time of maximum entanglement decrease by increasing the nonlinear coupling constant g. It is also shown that the survival time of maximum entanglement decreases when the difference between photon number of two modes A and B of the initial state is decreased.

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